FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C4

Paper J

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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C4 Paper J – Marking Guide

1. $x(x-2)=0,\ x=0,2\ \therefore$ crosses x-axis at (0, 0) and (2, 0)

volume $=\pi\int_0^2 (x^2-2x)^2\ \mathrm{d}x$ M1

$=\pi\int_0^2 (x^4-4x^3+4x^2)\ \mathrm{d}x$ A1

$=\pi[\frac{1}{5}x^5-x^4+\frac{4}{3}x^3]_0^2$ M1 A1

$=\pi\{(\frac{32}{5}-16+\frac{32}{3})-(0)\}=\frac{16}{15}\pi$ M1 A1 **(6)**

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2. $u=1-x^{\frac{1}{2}}\ \Rightarrow\ x=(1-u)^2,\ \dfrac{\mathrm{d}x}{\mathrm{d}u}=-2(1-u)=2u-2$ M1 A1

$I=\int \frac{1}{u}\times(2u-2)\ \mathrm{d}u=\int (2-\frac{2}{u})\ \mathrm{d}u$ A1

$=2u-2\ln|u|+c$ M1 A1

$=2(1-x^{\frac{1}{2}})-2\ln|1-x^{\frac{1}{2}}|+c$ A1 **(6)**

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3. *(a)* $4\cos 2x-\sec^2 y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ M1 A2

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=4\cos 2x\cos^2 y$ M1 A1

(b) grad $=4\times\frac{1}{2}\times\frac{1}{4}=\frac{1}{2}$ B1

$\therefore\ y-\frac{\pi}{3}=\frac{1}{2}(x-\frac{\pi}{6})$ M1

$y-\frac{\pi}{3}=\frac{1}{2}x-\frac{\pi}{12}$

$y=\frac{1}{2}x+\frac{\pi}{4}$ A1 **(8)**

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4. *(a)* $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2}at^{-\frac{1}{2}},\quad \dfrac{\mathrm{d}y}{\mathrm{d}t}=a(1-2t)$ M1

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{a(1-2t)}{\frac{1}{2}at^{-\frac{1}{2}}}=2\sqrt{t}\ (1-2t)$ M1 A1

(b) $y=0\ \Rightarrow\ t=0$ (at O) or 1 (at A) B1

$t=1,\ x=a,\ y=0,$ grad $=-2$ M1

$\therefore\ y-0=-2(x-a)$ A1

at $B,\ x=0\ \therefore\ y=2a$ M1

area $=\frac{1}{2}\times a\times 2a=a^2$ M1 A1 **(9)**

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5. *(a)* $\dfrac{\mathrm{d}y}{\mathrm{d}x}=k\sqrt{y}$

$\int y^{-\frac{1}{2}}\ \mathrm{d}y=\int k\ \mathrm{d}x$ M1

$2y^{\frac{1}{2}}=kx+c$ M1 A1

$(0, 4)\ \Rightarrow\ 4=c$ M1

$\therefore\ 2\sqrt{y}=kx+4$ A1

(b) $(2, 9)\ \Rightarrow\ 6=2k+4,\quad k=1$ M1 A1

$\therefore\ 2\sqrt{y}=x+4,\quad \sqrt{y}=\frac{1}{2}(x+4)$ M1

$y=\frac{1}{4}(x+4)^2$ A1 **(9)**

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6. *(a)* let radius = r, $\therefore \tan 30° = \frac{1}{\sqrt{3}} = \frac{r}{h}$, $r = \frac{h}{\sqrt{3}}$ M1

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h \times \frac{h^2}{3} = \frac{1}{9}\pi h^3$ M1 A1

(b) *(i)* $\frac{dV}{dt} = 120$, $\frac{dV}{dh} = \frac{1}{3}\pi h^2$ B1

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$, $120 = \frac{1}{3}\pi h^2 \frac{dh}{dt}$, $\frac{dh}{dt} = \frac{360}{\pi h^2}$ M1 A1

when $h = 6$, $\frac{dh}{dt} = 3.18$ cm s^{-1} (2dp) M1 A1

(ii) $V = 8 \times 120 = 960 = \frac{1}{9}\pi h^3$ $\therefore h = \sqrt[3]{\frac{9\times960}{\pi}} = 14.011$ M1

$\therefore \frac{dh}{dt} = 0.58$ cm s^{-1} (2dp) A1 **(10)**

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7. *(a)* $\overrightarrow{AB} = \begin{pmatrix}-3\\6\\1\end{pmatrix} - \begin{pmatrix}-4\\1\\3\end{pmatrix} = \begin{pmatrix}1\\5\\-2\end{pmatrix}$ $\therefore$ $\mathbf{r} = \begin{pmatrix}-4\\1\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\5\\-2\end{pmatrix}$ M1 A1

(b) $-4 + \lambda = 3 + 2\mu$ (1)
$1 + 5\lambda = -7 - 3\mu$ (2)
$3 - 2\lambda = 9 + \mu$ (3) B1
$2 \times (1) + (3)$: $-5 = 15 + 5\mu$, $\mu = -4$, $\lambda = -1$ M1 A1
sub. (2): $1 - 5 = -7 + 12$, not true $\therefore$ do not intersect M1 A1

(c) $\overrightarrow{OC} = \begin{pmatrix}3+2\mu\\-7-3\mu\\9+\mu\end{pmatrix}$, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix}6+2\mu\\-13-3\mu\\8+\mu\end{pmatrix}$ M1 A1

$\therefore \begin{pmatrix}1\\5\\-2\end{pmatrix} \cdot \begin{pmatrix}6+2\mu\\-13-3\mu\\8+\mu\end{pmatrix} = 0$, $6 + 2\mu - 65 - 15\mu - 16 - 2\mu = 0$ M1 A1

$\mu = -5$ $\therefore \overrightarrow{OC} = \begin{pmatrix}-7\\8\\4\end{pmatrix}$ M1 A1 **(13)**

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8. *(a)* $x(3x - 7) \equiv A(1 - x)(1 - 3x) + B(1 - 3x) + C(1 - x)$ M1
$x = 1 \Rightarrow -4 = -2B \Rightarrow B = 2$ A1
$x = \frac{1}{3} \Rightarrow -2 = \frac{2}{3}C \Rightarrow C = -3$ A1
coeffs x^2 $\Rightarrow 3 = 3A \Rightarrow A = 1$ A1

(b) $= \int_0^{\frac{1}{4}} (1 + \frac{2}{1-x} - \frac{3}{1-3x})\, dx = [x - 2\ln|1 - x| + \ln|1 - 3x|]_0^{\frac{1}{4}}$ M1 A1

$= (\frac{1}{4} - 2\ln\frac{3}{4} + \ln\frac{1}{4}) - (0)$ M1

$= \frac{1}{4} + \ln\frac{16}{9} + \ln\frac{1}{4} = \frac{1}{4} + \ln\frac{4}{9}$ M1 A1

(c) $f(x) = 1 + 2(1 - x)^{-1} - 3(1 - 3x)^{-1}$
$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots$ B1
$(1 - 3x)^{-1} = 1 + 3x + (3x)^2 + (3x)^3 + \ldots = 1 + 3x + 9x^2 + 27x^3 + \ldots$ M1 A1
$\therefore$ $f(x) = 1 + 2(1 + x + x^2 + x^3 + \ldots) - 3(1 + 3x + 9x^2 + 27x^3 + \ldots)$ M1
$= -7x - 25x^2 - 79x^3 + \ldots$ A1 **(14)**

Total **(75)**

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Performance Record – C4 Paper J

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	integration	integration	differentiation	parametric equations	differential equation	connected rates	vectors	partial fractions, binomial series	
Marks	6	6	8	9	9	10	13	14	75
Student									

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